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REMODELLING OF ADAPTIVE STRUCTURES DESIGNED FOR IMPACT LOADS

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Abstract. *The methodology (based on the so-called Dynamic Virtual Distortion Method) of redesign of structures exposed to impact loading is presented in the work. Minimization of material volume and accelerations of structural response can be chosen as the objective functions for optimal design of structures adaptating to impact loads. The cross-sections of structural members as well as stress levels triggering plastic-like behavior and initial prestressing can be the design parameters. A general formulation of this problem, as well as particular cases, are discussed.*

1 INTRODUCTION

Motivation for the undertaken research is to respond to requirements for high impact energy absorption e.g. in the structures exposed to the risk of extreme blast, light, thin wall tanks with high impact protection, vehicles with high crashworthiness, protective barriers, etc. Typically, the suggested solutions focus on the design of passive energy absorbing systems. These systems are frequently based on the aluminum and/or steel honeycomb packages characterized by a high ratio of specific energy absorption. However high is the energy absorption capacity of such elements, they still remain highly redundant structural members, which do not carry any load in the actual operation of a given structure. In addition, passive energy absorbers are designed to work effectively in pre-defined impact scenarios. For example, the frontal honeycomb cushions are very effective during a symmetric axial crash of colliding cars, but are completely useless in other types of crash loading. Consequently, distinct and sometimes completely independent systems must be developed for specific collision scenarios.

In contrast to the standard passive systems, the proposed approach focuses on active adaptation of energy absorbing structures (equipped with a sensor system detecting impact in advance and controllable semi-active dissipaters, so called structural fuses) with a high ability of adaptation to extreme overloading. The concept formulation and first numerical analysis are based on the previously published paper [12]. Various formulations of crashworthiness-based structural design problem are presented in papers [1]-[9], while the adaptive crashworthiness concept has been first proposed in [10]-[11]. The optimal design methodology proposed below combines sensitivity analysis with the redesign process, allowing optimal redistribution of material as well as stress limit control in structural fuses. It is assumed that this “smart” devices are able to release structural connections in a controlled way, triggering plastic-like distortions mimicking elasto-plastic behaviour shown in Fig.2.

The objective of this presentation is to propose numerical tools for efficient redemodelling of structures under impact loads.

Taking into account the following definitions:

E^u – maximal expected impact energy

σ^u – yield stress level for ideal elasto-plastic material used to built the structure

β^u – maximal allowed plastic-like distortion to be generated in structural fuses.

the following optimization problem can be considered.

$$\min \mathrm{V} = \min \sum_{\mathrm{i}} \mu_i A_i^{\prime} l_i \tag{1.1}$$

subject to constraints:

$$\begin{gathered} \left|\beta_{\mathrm{i}}^0(t)\right| \leq \beta^u \\ \left|\sigma_{\mathrm{i}}(t)\right| \leq \sigma^* \leq \sigma^{\mathrm{u}} \\ \beta_{\mathrm{i}}^0(t)\sigma_i(t) \geq 0 \end{gathered} \tag{1.2}$$

where l_i denotes the length of the member i and stresses $\sigma_i(t)$ depend on the maximal expected impact load $I(m,v)$ and the control parameters: μ_i, σ_i^*, $\beta_i^{0'}$, what will be discussed in the next section. The constraint $(1.2)^3$ describes the condition of dissipative character of plastic-like distortions generation

2 VDM BASED DYNAMIC REMODELING OF ADAPTIVE STRUCTURE

In this chapter we will formulate the VDM based description of the dynamic response of elasto-plastic truss structure. Applying discretized time description, the evolution of strains and stresses (with respect to initial cross-sections) can be expressed as follows:

$$\varepsilon_i(t) = \varepsilon_i^L(t) + \sum_{\tau \le t}\sum_j D_{ij}(t-\tau)\cdot\varepsilon_j^0(\tau) + \sum_{\tau \le t}\sum_k D_{ik}(t-\tau)\cdot\beta_k^0(\tau) \tag{2.1}$$

$$\begin{aligned} \sigma_i^{'}(t) &= E_i\left(\varepsilon_i(t) - \varepsilon_i^0(t) - \beta_i^0(t)\right) \\ \sigma_i^{'}(t) &= E_i\left[\varepsilon_i^L(t) + \sum_{\tau \le t}\sum_j D_{ij}(t-\tau)\cdot\varepsilon_j^0(\tau) - \varepsilon_i^0(t) + \sum_{\tau \le t}\sum_k D_{ik}(t-\tau)\cdot\beta_k^0(\tau) - \beta_i^0(t)\right] \end{aligned} \tag{2.2}$$

where so called dynamic influence matrices $D_{ij}(t-\tau)$ describe the strain evolution caused in the truss element member *i* and in time instance *t*, due to unit virtual distortion impulse generated in member j in the time instant τ. The vector $\varepsilon_i^L(t)$ denotes the strain evolution due to external loads applied to the elastic structure with initial material distribution (unmodified cross-sections of members), $\varepsilon_i^0(t)$ denotes virtual distortions responsible for modification of design variables and $\beta_i^0(t)$ describes plastic-like distortions. Note that matrix **D** stores information about the properties of the entire structure (including boundary conditions) and describes dynamic (not static) structural response to locally generated impulse of virtual distortion. Note also that it was assumed here the influence of local modifications of design variables on the stiffness matrix only. The full analysis taking into account the influence of virtual distrotions $\varepsilon_i^0(t)$ on both, the stiffness as well as the mass matrices is more complicated and will be discussed in separate section. From now on, we assume that small Latin index *j* runs through all modified members, and small Latin index *k* runs through all palstified elements.

In order to take into account elasto-plastic structural behaviour, let us use the bilinear constitutive model hardening (Fig. 1), given by the equation (2.3)

$$\sigma_i(t) - \sigma_i^* = \gamma_i E_i\left(\varepsilon_i(t) - \varepsilon_i^*\right) \tag{2.3}$$

where σ_i^* denotes plastic yield stress, γ_i denotes hardening parameter and E_i denotes Young's modulus.

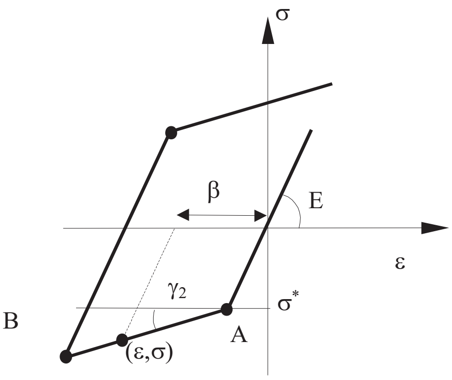

Fig. 1. Piece-wise linear constitutive relation for the adaptive structural member

Now, when we substitute stress (2.2) and strain (2.1) evolution in time to the formula (2.3) we obtain the following equations:

$$\beta_i^0(t) = (1-\gamma_i)\left(\varepsilon_i^L(t) - \varepsilon_i^*\right) + (1-\gamma_i)\sum_{\tau\le t}\sum_j D_{ij}^D(t-\tau)\cdot\varepsilon_j^0(\tau) + (1-\gamma_i)\sum_{\tau\le t}\sum_k D_{ik}^H(t-\tau)\cdot\beta_k^0(\tau) \tag{2.4}$$

Taking advantage of two expressions for the internal forces applied to the so called distorted (2.5) (with modification of material distribution modeled through virtual distortions) and modified (2.6) (with redesigned cross-sections form A to A', without imposing virtual distortions) structure:

$$P_i(t) = E_i A_i\left(\varepsilon_i(t) - \varepsilon_i^0(t) - \beta_i^0(t)\right) \tag{2.5}$$

$$P_i(t) = E_i A_i^{'}\left(\varepsilon_i(t) - \beta_i^0(t)\right) \tag{2.6}$$

A formula combining components $\varepsilon_i^0(t)$ and $\beta_i^0(t)$ can be derived, where these components are non zero only for distorted and or plastified elements.

If we assume that forces and strains in both structures: *distorted* (2.5) and *modified* (2.6) are the same, the modifications simulated with virtual distortion can be combined with these distortions through the flowing formula:

$$\varepsilon_i^0(t) = (1-\mu_i)\left(\varepsilon_i(t) - \beta_i^0(t)\right) \tag{2.7}$$

where $\varepsilon_i(t)$ describes strain in member *i* in time *t*, while $\mu_i = A_i / A_i^{'}$ denotes ratio of the new cross-section to the initial one. Parameter $\mu_i \in \langle 0,1\rangle$ specifies size of modification of cross-sections in element *i*. If $\mu_i = 1$ that means that in element *i* the cross-section does not change, and if $\mu_i = 0$ that means that element *i* can be neglected in the analysis.

The formula (2.7) can be rewritten in the following form (2.8):

$$\mu_i = \frac{A_i}{A_i^{'}} = \frac{\varepsilon_i(t) - \varepsilon_i^0(t) - \beta_i^0(t)}{\varepsilon_i(t) - \beta_i^0(t)} \tag{2.8}$$

Now let us substitute strain evolution in time (2.1) to formula (2.8) getting the following set of equations:

$$\varepsilon_i^0(t) = (1-\mu_i)\varepsilon_i^L(t) + \sum_{\tau\le t}\sum_j D_{ij}^D(t-\tau)\cdot\varepsilon_j^0(\tau) + \sum_{\tau\le t}\sum_k D_{ik}^H(t-\tau)\cdot\beta_k^0(\tau) - \beta_i^0(t) \tag{2.9}$$

Note that the equations (2.4) are not dependant on the virtual distortions responsible for modification of design variables in time t $\varepsilon_i^0(t)$, but only on the distortions in previous time steps $\varepsilon(\tau)$ $\tau < t$ because of the assumption (2.10).

$$D_{ij}(0) = 0 \tag{2.10}$$

Therefore, the plastic-like distortions $\beta_i^0(t)$ should be calculated first in each time step of the algorithm.
Equations (2.4) and (2.9) need only computation of the right-hand side expressions, and we need not solve the coupled sets of equations.
Formulas (2.4) and (2.9) allow us to compute the virtual distortions' development in time, modeling both: assumed remodeling of material distribution as well as adapted plastic-like stress limits.

If there is no plasticity in our problem, then plastic-like distortions β_i^0 are equal to zero and the equation (2.9) takes the following form:

$$\varepsilon_i^0(t) = (1-\mu_i)\varepsilon_i^L(t) + \sum_{\tau\le t}\sum_j D_{ij}(t-\tau)\cdot\varepsilon_j^0(\tau) \tag{2.11}$$

Analogously, if there is no remodeling, distortions ε_i^0 are equal to zero (the parameter μ_i is equal to one) and equation (2.4), determining plastic like distortions development takes the following form:

$$\beta_i^0(t) = (1-\gamma_i)\left(\varepsilon_i^L(t) - \varepsilon_i^*\right) + (1-\gamma_i)\sum_{\tau\le t}\sum_k D_{ik}^H(t-\tau)\cdot\beta_k^0(\tau) \tag{2.12}$$

To prove that the VDM method gives the same solutions as commercial programs let us compare results for the simple truss structure shown below with the structural response determined with ANSYS

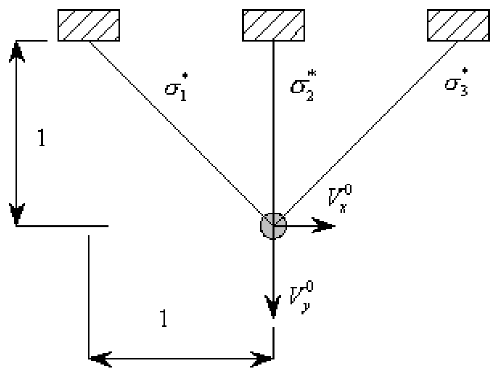

Fig. 1. Testing example truss structure. (Young modulus E_i=2.1e11 [Pa], cross-sections A_i=1e-4 [m], density ρ_i=7800 [kg/m^3])

All elements have different yield stress limits σ_i^* as well as parameters responsible for modification of design variables μ_i.

σ_1^* = 8e7 [Pa], σ_2^* = 4e7 [Pa] σ_3^* = 6e7 [Pa]

$\mu_2 = 0{,}7$ $\mu_1 = 0{,}5$ $\mu_3 = 0{,}9$

In the lower node, concentrated mass 20 [kg] is added, together with the following initial condition (modeling with external object):

$V_x^0 = 3$ [m/s], $V_y^0 = 5$ [m/s]

On the graphs shown below the comparison of strain (fig. 4), stress (fig. 5) and plastic distortion (fig. 6) development for the first and the second element, respectively is demonstrated.

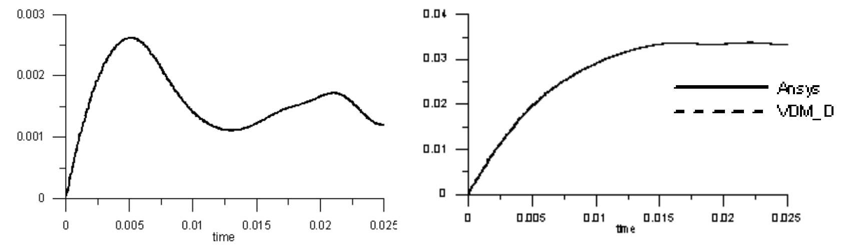

Fig. 2. Strain evolution in time for elements: a) left element, b) central element

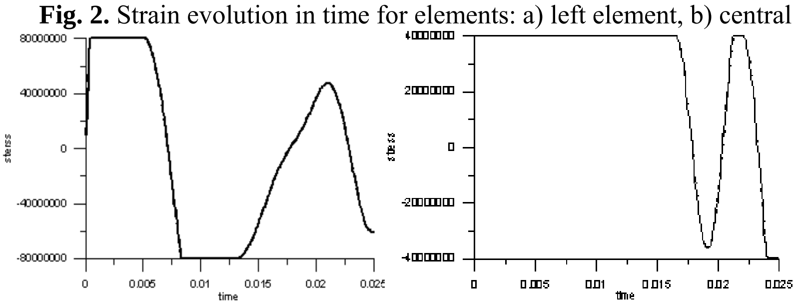

Fig. 3. Stress evolution in time for elements: a) left element, b)central element

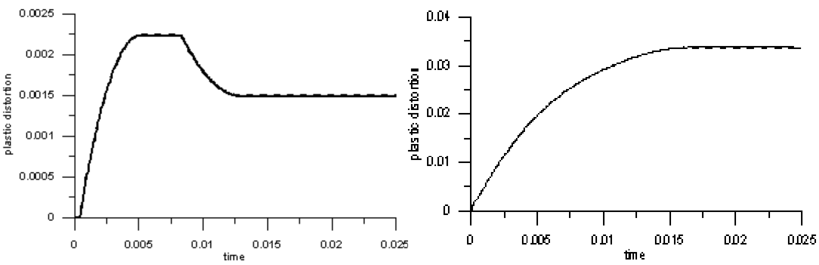

Fig. 4. Plastic distortion evolution in time for elements: a) left element, b) central element

Note, that only modification of the stiffness matrix (due to remodeling) has been taken into account in the above formulas. Analogous modification of the mass matrix has to be added in order to describe the complex remodeling phenomena. However, it was decided to keep this presentation simpler without additional complication of formulas.

3 VDM BASED REDESIGN OF MASS DISTRIBUTION

Let us now focus on the problem of modeling of mass redistribution in the structure applying analogous methodology to the VDM approach described above. In this case, however, we are going to use *virtual forces* rather than *virtual distortions* to model modified inertial forces due to modification of mass distribution. Therefore, the influence matrix D will be defined differently in this case, determining structural response to unit impulse forces applied in structural nodes. Following this *Virtual Force Method* (VFM) approach let us apply the following description of evolution of displacements, velocities and accelerations:

$$\begin{aligned} u_i(t) &= u_i^L(t) + \sum_{\tau\le t}\sum_m D_{im}(t-\tau)\, p_m^0(\tau) \\ \dot{u}_i(t) &= \dot{u}_i^L(t) + \sum_{\tau\le t}\sum_m D_{im}(t-\tau)\, \dot{p}_m^0(\tau) \\ \ddot{u}_i(t) &= \ddot{u}_i^L(t) + \sum_{\tau\le t}\sum_m D_{im}(t-\tau)\, \ddot{p}_m^0(\tau) \end{aligned} \tag{3.1}$$

where the dynamic influence matrix $D_{im}(t-\tau)$ describes the displacement evolution caused in the truss node *i* and in time instance *t*, due to unit virtual force generated in node *m* of the originally configured structure (assuming lumped mass matrix) in the time instant τ. The vector $u_i^L(t)$ denotes the displacement evolution due to external loads applied also to the unmodified structure, $p_i^0(t)$ denotes virtual forces modeling modification of mass distribution. Note that the matrix **D** stores information about the properties of the entire structure (including boundary conditions) and describes dynamic (not static) structural response to locally generated impulse of virtual force. From now on, we assume that small Latin index *m* runs through all modified nodes.

The equation of motion for unmodified structure, with unchanged mass **M** distribution and *virtual forces* modeling mass redistribution takes the form (3.2). On the other hand, the equation of motion for the modified structure, with modified mass **M'** distribution takes the form (3.3).

$$M\ddot{u}_i(t) + Ku_i(t) - p_i^0(t) = F(t) \tag{3.2}$$

$$M'\ddot{u}_i(t) + Ku_i(t) = F(t) \tag{3.3}$$

Assuming that forces $F(t)$ and displacements $u_i(t)$ in both structures: the *modified* one (3.3) and the *modeled* (3.2) are the same, the mass modifications and modeling them *virtual forces* can be combined through the flowing formula:

$$\left(M - M'\right)\ddot{u}_i(t) = \Delta M\ddot{u}_i(t) = p_i^0(t) \text{ or } M\ddot{u}_i(t)\left(1-\mu_i^M\right) = p_i^0(t) \tag{3.4}$$

where accelerations of displacements $u_i(t)$ can be presented in the form (3.1), while $\mu_i = M_i^{'} / M_i$ denotes the ratio of the new mass to the initial one. Parameter $\mu_i \in \langle 0, M^{\max} \rangle$ specifies the intensity of mass modification in node *i*. If $\mu_i = 1$ that means that the mass does not change in the node *i*, and if $\mu_i = 0$ that means that mass of the node *i* vanishes. The value $M^{\max}$ denotes the maximal acceptable mass concentrated in one node..

Substituting $(3.1)^3$ to (3.4) the following formula allowing determination of the virtual force evolution can be derived:

$$p_i^0(t) - M\left(1-\mu_i^M\right) D(0)\, \ddot{p}(t) = M\left(1-\mu_i^M\right)\left[\ddot{u}_i^L(t) + \sum_{\tau<t}\sum_m D_{im}(t-\tau)\, \ddot{p}_m^0(\tau)\right] \tag{3.5}$$

To prove that the proposed method gives the same solutions as classical methods let us compare results for the truss structure shown below.

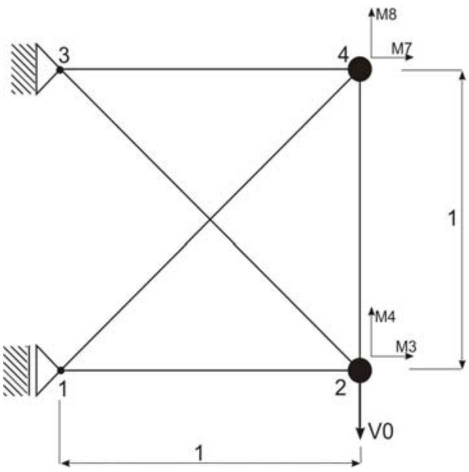

Fig. 5. Testing example truss structure. (Young modulus E_i=2.1e11 [Pa], cross-sections A_i=1e-4 [m], density ρ_i=7800 [kg/m^3])

All elements have the same material parameters, and in nodes 2 and 4 concentrated mass 100 [kg] is added, in node 2 initial condition (modeling with external object): V_y^0 = -10 [m/s]. In node 2 μ_3^M and μ_4^M is equalt 0.8 and in node 4 μ_7^M and μ_8^M is equalt 0.4.

On the graphs shown below the comparison of displacement, velocity and acceleration development for the node 2 in *y* direction, respectively is demonstrated.

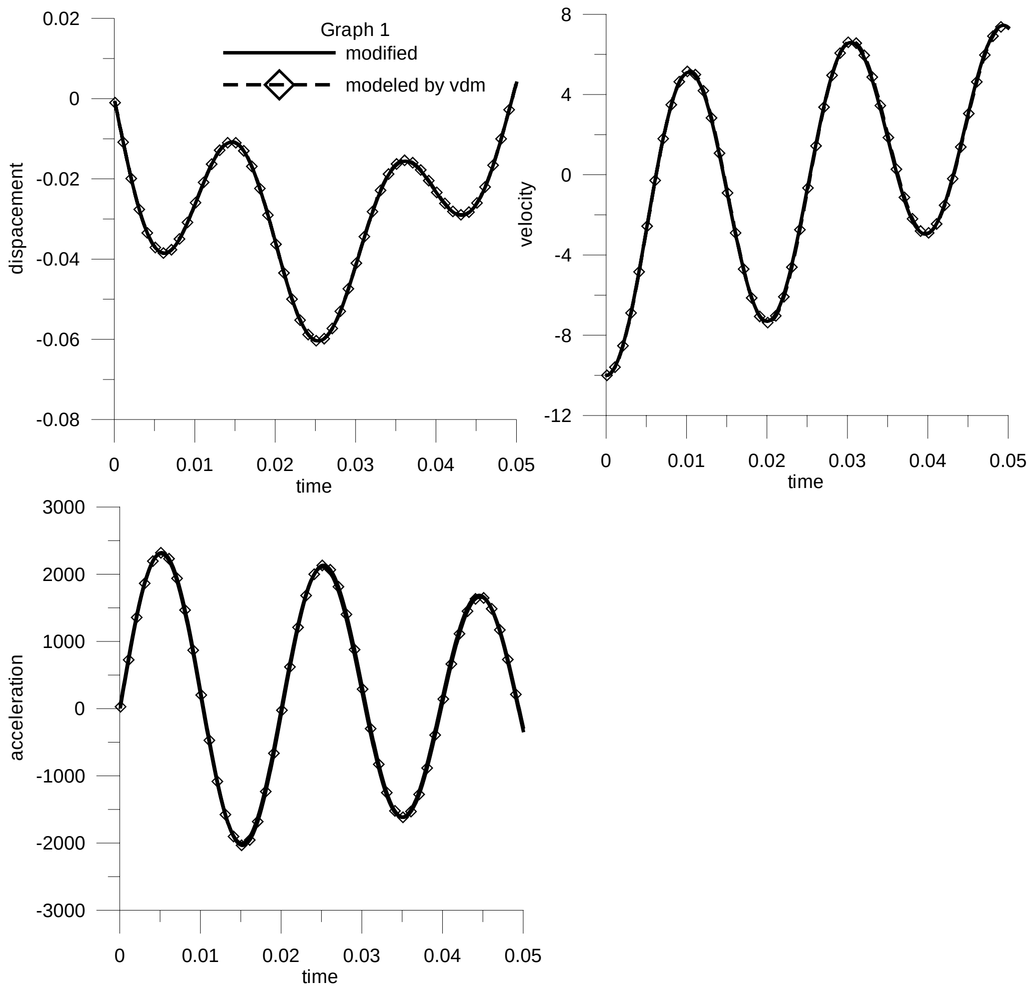

Fig. 6. Displacement, velocity and acceleration in time, for modified (with changed mass) and modeled using VDM structure.

4 CONCLUSION

The methodology (based on the so-called Dynamic Virtual Distortion Method) of the design of structures exposed to impact loading is presented in the work. Minimization of material volume is chosen as the objective functions for optimal design of structures adaptating to impact loads. The cross-sections of structural members as well as stress levels triggering plastic-like behavior and initial prestressing can be the design parameters.

The paper demonstrates the effectiveness of the proposed concept. The yield stress level adaptation to the applied load has significant influence on the intensity of impact energy dissipation.

5 ACKNOWLEDGEMENT

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